Discussion

Comment on "Craze growth and craze interactions"

In a recent paper by Mills [1], attention was drawn to shortcomings in the Argon-Salama [2] and Verheulpen-Heymans-Bauwens [3,4] models for craze growth and it appears that the following points require clarification.

Firstly, there appears to be some misunderstanding as to the requirements of the elastic stress analysis. Whatever the method used, material outside the boundary is considered to be linearly elastic. This means the boundary must be chosen so as to exclude anelastic behaviour; in practice, crazing is generally investigated at average stresses considerably below yield, therefore material outside the physical craze can be considered as linearelastic, so the elastic boundary can be taken coincident with the physical craze boundary. It should be kept in mind that in all events, the point at which the craze ends is open to question. Referring to Fig. 1, which is a real stress distribution along a crack-tip DZ in PC [5], the body could in this case be taken as the region where $\sigma < \bar{\sigma}_{c}$, the tip reaching to the inflexion point in the stress distribution.

When deriving growth laws from a stress distribution the following points should be kept in mind.

(1) Contrary to crack behaviour, a craze is loadbearing, and therefore strain energy release rate criteria can be extremely misleading if an approximate stress distribution is used, as in the Argon-



Figure 1 Surface stress along a crack-tip DZ in PC measured by Donald and Kramer [5]. Curve redrawn. $\tilde{\sigma}_c$: average stress along crack-tip craze.

Salama [2] and Verheulpen-Heymans-Bauwens [4] models, which both use a Dugdale-type distribution with a constant normal stress, σ_e , along the craze tip and σ_c along the craze body. Craze growth is necessarily energetically favourable if the material presents an instability at yield, since in this case one will have $\sigma_e > \sigma_{\infty}$ and $\sigma_c < \sigma_{\infty}$, but not much more information than this can be obtained from energy release rate considerations with no prior knowledge of craze material behaviour.

(2) If craze growth is thought of as a kinetic process, this process requires three steps: craze tip yielding, fibrillation (both leading to lengthening) and thickening by drawing in material from the walls and by creep, (the faster of these two processes being dominant). In solvent crazing one should add a fourth process, diffusion. The ratedetermining step is the slowest; in PC crazes it was clear that the rate-controlling step was craze thickening [3, 6], mainly by drawing-in, and that growth rates were independent of conditions at the craze tip, in particular of the yield stress [7]. (Incidentally the tip stress, σ_e , probably does not remain constant, since the growth rate decreases with time; as long as $r \ll a$ this does not affect the analysis.) As stated previously [6], this model is not at all in contradiction with the Argon-Salama model [2], which is complementary, since it gives a value for craze fibril diameters which is compatible with our own results. The fact that the Argon-Salama model gives quantitative agreement with observed growth rates in PMMA and PS simply implies that in these materials the ratecontrolling step is propagation at the craze tip. The difference in behaviour between PMMA or PS, and PC is probably connected with different behaviour of the entanglement network, giving a higher natural draw ratio in PS than in PC [8], allowing PC crazes to bear a higher load, σ_c , than PS crazes [9], which therefore exhibit somewhat "crack-like" behaviour in that growth rates depend more strongly on craze-tip conditions than on craze body conditions. The latter should not, however, be overlooked since fracture will initiate in the oldest part of the midrib.

The statement that the two models are incompatible is therefore not founded, on the contrary. Also, it is not entirely true to say that they are unable to predict the effect of interactions. Both models are mainly concerned with what goes on inside the craze, whereas interactions only affect the elastic stress field analysis outside the craze.

Next, to reply to the comment that in [4] the craze initiation problem was not considered: it was found in [3] that craze initiation kinetics were quite different from craze growth kinetics, which is to be expected since conditions prevailing during growth ($e \ll a$, $r \ll a$ and σ_c close to σ_{∞}) cannot be expected to obtain during the initiation phase at a region of local weakness which could be approximated, not by a thin slit but by a weak spherical or elliptical inclusion. Argon and Salama [2] also considered the growth problem as being independent of the initiation problem investigated by Argon and Hannoosh [10]. It was felt in [3] that since in brittle materials the time to failure is mainly taken up by craze growth, this problem was more relevant than initiation to fracture.

In his discussion, Mills questions the validity of the assumption [11] that drawing-in of fresh material occurs by a similar mechanism to neck propagation, since creep strain kinetics are very different from craze growth kinetics in PVC, and since brittle fracture occurs before necking in this material. The first objection does not hold, firstly because craze thickening occurs mainly by drawingin and not by creep [6, 12] so that low-strain creep is irrelevant; and secondly because if craze thickening is not the rate-limiting step, craze growth kinetics will necessarily be different from neckpropagation kinetics. Also, results on bulk specimens cannot be used as such, but have to be scaled down to craze fibril dimension [11]. The second objection does not hold because brittle failure and necking are two competitive processes: necking could occur if brittle failure could be suppressed, but brittle failure is due precisely to growth and fracture of crazes. This is an experimental difficulty in all materials with a high yield stress. Even in PC it was not found possible to propagate a neck in annealed samples [7]. Indirect evidence that craze thickening occurs by a cold-drawing mechanism has been found in PMMA and PC [13].

Lastly, Mills questions the validity of a two-

dimensional analysis. It is, of course, clear that a two-dimensional analysis is physically unrealistic, as are assumptions of constant stress along the tip and body: there is no sudden transition at the tip and between the tip and body; however, this analysis gives quite adequate results, as shown in a comparison of surface displacements obtained from this analysis with displacements obtained in a finite element analysis by Bevan [14]. A twodimensional analysis also offers the great advantage over a three-dimensional analysis of being analytically far simpler and more manageable, as Mills realized since he also used a two-dimensional analysis. In connection with this analysis, Mills found an error in Equation A24 of [4]. In fact, Equation A24 is correct, but the sign before Im in Equation A25 and the sign of Equation A26 should be a plus, not a minus.

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